Real Time Solver for Model Predictive Control

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Abstract

Model Predictive Control (MPC) has begun to receive significant attention as a tool in real-time solver applications, as the betterment of technology and algorithms have allowed it to have a much faster operation time. This work explores the use of Lemke's Algorithm (Scheme 1) in order to solve the Linear Complementarity Problem (LCP) that is derived from a specific MPC formulation. The conversion of MPC to LCP used by the proposed algorithm is shown in the "Methodology" section. The computation time of the proposed algorithm shows the promise it has as an efficient solver for Model Predictive Control.

Introduction

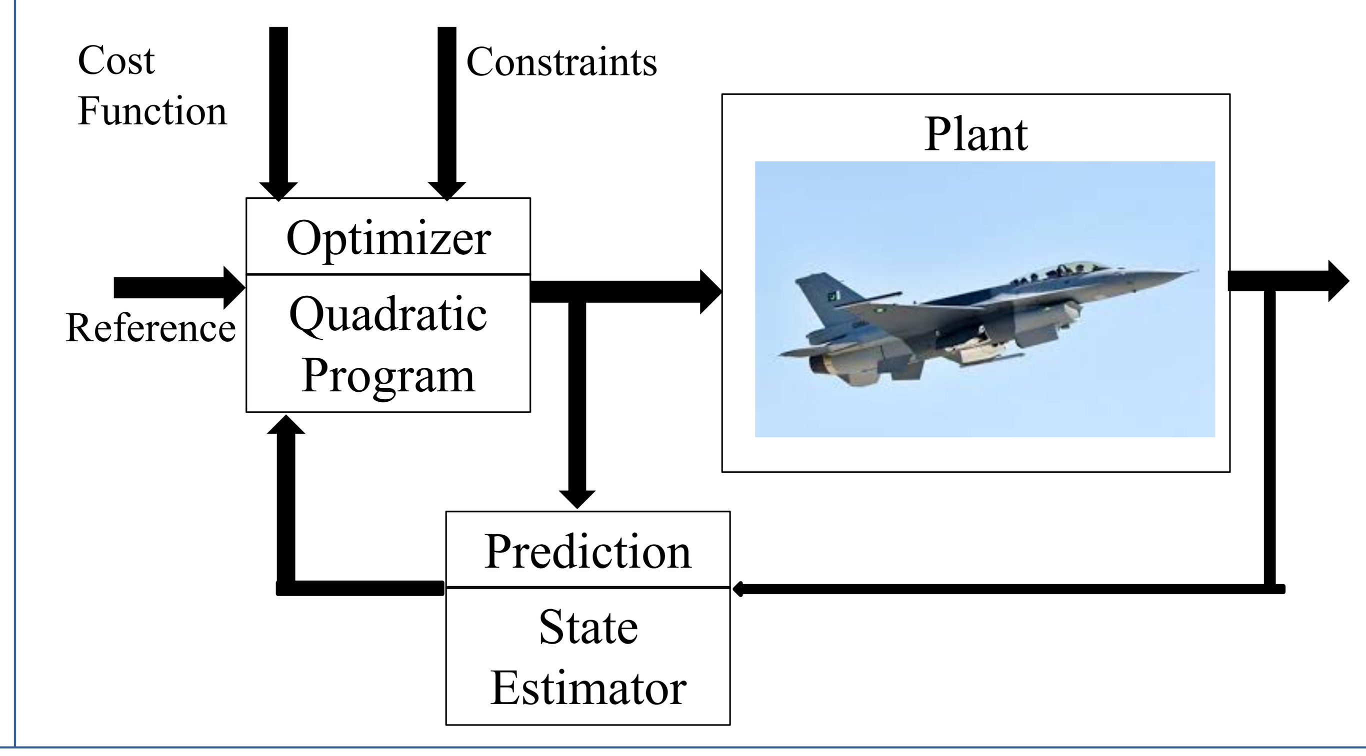
- Previously, MPC could only be applied in with systems operated in seconds or minutes. However, due to the use of online optimization similar to the methods used in this research have allowed MPC problems to be solved much faster [3]
- Lemke's Method is a type of pivoting function which drives a specific variable to 0 unless it is blocked by a different variable. It then selects a new driving variable and repeats until the desired result is achieved.
- The solver discussed was developed throughout the research period and proves that the use of Lemke's Method in MPC solvers is efficient.

Conclusions

- The research shown has proven that Lemke's Method can be used to build a reliable solver that is faster than the available general solvers.
- The QPSolver discussed was also adapted to solve Non-Condensed Model Predictive Control in an efficient manner
- Future work includes making additions to the current algorithm to also accommodate the Condensed MPC formulation, as well as testing various LCP solving techniques against Lemke's to find the one with the best results.
- Currently, the solver is being implemented onto a Quanser 2Dof-Helicopter, to evaluate its ability to operate in real time.

Model Predictive Control Structure

- Model Predictive Control (MPC) is an advanced control strategy that involves online computation of a dynamic program during each plant update.
- MPC consists of an optimizer and a state estimator, which allows prediction and optimization of states and input values over a future and moving horizon.
- The huge online computational burden has hitherto limited MPC to process control but recent advancements in both hardware and software have brought MPC to the realm of fast systems such as in aerospace systems.
- Within aerospace engineering, examples of systems MPC can control include stabilization, docking, and self-flying mechanisms.



Methodology

MPC Problem Formulation:

$$J = \frac{1}{2}\left(x_N^T P_N x_N + \sum_{t=k+1}^{k+N} x_t^T Q_t x_t + \sum_{t=k}^{k+Nu-1} u_t^T R_t u_t\right)$$

$$\text{Subject to: } x_{t+k+1} = Ax_{t+k} + Bu_{t+k}$$
$$\text{for } k = 0,1, Np-1\,;$$
$$x \in \mathbb{X}, u \in \mathbb{U}$$

- The MPC is formulated as a problem of optimizing some cost function subject to the plant prediction and associated constraints on both input and states.
- A common formulation is the constrained linear quadratic regulator shown above where Q and R are weighting matrices.

Quadratic Problem:

$$\min \frac{1}{2} p^T Hp + p^T f$$
$$\text{subject to: } Ep = e\,; Cp \geq b$$

- The MPC can be reformulated into a quadratic programming (QP) problem that must be solved every time step.
- Usually the Hessian matrix H, and the constraints matrices E and C are fixed and depend directly on the MPC problem data. The vectors f and e are usually time varying.

Mixed Linear Complementarity Problem:

$$w = Mz + q$$
$$\text{subject to: } z \geq 0\,;\ z^T(Mz + q) = 0$$

- Using the Karush Kuhn Tucker (KKT) optimality conditions, the QP can in turn be translated into a Mixed Linear Complementarity Problem (MLCP).
- The fixed matrix M encapsulates the problem data and determines the solvability of the MLCP while the vector q is time varying.
- Using a pivot-based algorithm, the MLCP can solved completely over a fixed number of pivot operations.

Results

- The QP algorithm that utilizes Lemke's Method was tested against other well-known solvers, namely Matlab's 'quadprog' and 'mpcActiveSetSolver' functions.
- The computation times were measured using the Matlab 'timeit' function, which runs the algorithm many times and outputs the median computation time. This was done ten times and the average of the times are displayed in table 1.
- The following variables were inputted into the algorithms:
  - $H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; f = \begin{bmatrix} 3 \\ 4 \end{bmatrix}; C = \begin{bmatrix} -1 & -3 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}; b = \begin{bmatrix} -15 \\ 100 \\ 80 \end{bmatrix}$
- As shown in table 1, the solver that uses the Lemke Method in its algorithm operates faster than both solvers it was compared to

Table 1. Comparison of the average median computation time of different QP solvers

	Computation Time (ms)
QPSolver	0.1445
Quadprog	5.74
mpcActiveSetSolver	0.4281

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