Using Linear Matrix Inequalities to Optimize Anti-windup Gain for Convex Systems

Francis Moran, Dr. Ambrose Adegbege
The College of New Jersey School of Engineering
New Jersey Space Grant Consortium

Introduction

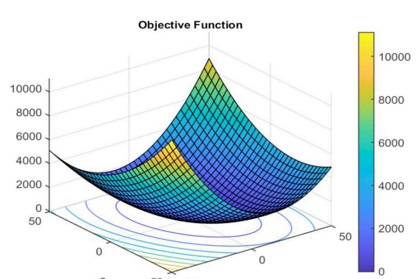
This research sought out to determine a method for finding an optimal anti-windup gain for a convex system. The combination of two past works led to this method. The first publication, Static Anti-Windup Design, used a linear matrix inequality, or LMI, to determine an anti-windup gain for a nonlinear system. This LMI guaranteed regional stability paired with performance. The second paper, Saddle Point Convergence of Constrained Primal-Dual Dynamics, showed how a convex primal dual system can be implemented with control. This research bridges the gap between these works.

Convex Optimization Problem

We consider convex optimization problem of the form:

$$\min_x f(x)$$
$$\text{Subject to } g(x) = Cx - e \leq 0$$

- Convex objective function *f(x)*
- Linear inequality constraints *g(x)* ≤ 0



Classical Primal Dual Dynamics

Classical primal dual dynamic for solving convex optimization problems comprises both gradient descent flow in the primal variable x and gradient ascent flow in the Lagrangian (dual)variable λ towards the optimal solution.

$$\dot{x} = -\nabla f(x) + C^T\lambda$$
$$\dot{\lambda} = Cx - e$$

Methodology

The approach adopted here is to reformulate the primal-dual dynamics as a combination of three parts:

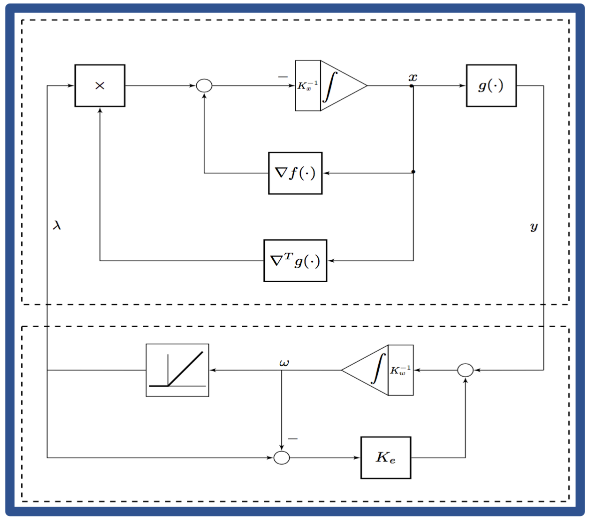
- An optimization plant corresponding to the gradient descent flow
- An optimization controller corresponding to the gradient ascent flow and
- An antiwindup augmentation that allows for affecting the behavior of the solver to a desired performance level.

The main task then is the determination of an appropriate antiwindup gain that encapsulates such performance specification.

The ensuing dynamical solver takes the form:

$$\dot{x} = -\nabla f(x) + Bv; \quad y = Cx$$
$$\dot{w} = Aw + B(y - e) + \xi; \quad \lambda = Cw + D(y - e)$$
$$u = -\lambda; \quad v = \phi(u); \quad \xi = f(x) = K_e\,(u - v)$$

Where K_e is the antiwndup gain to be determined.



Primal-dual dyanmics with antiwindup augmentation

Anti-Windup Gain Computation

Below is the reduced LMI for convex systems. This LMI contains LMI variables, parameters derived from the system, and anti-windup gain. This LMI is solved using MATLAB, and the result is used in simulation.

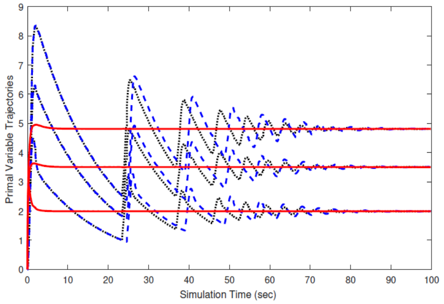
```
Q = sdpvar(n,n,'symmetric');
M = sdpvar(nu,nu,'diag');
X = sdpvar(size(Bsigma,2),nu);
gamma = sdpvar(1,1);

LMI = [[A*Q + Q*A',         Bw,             Bq*M + Bsigma*X + Q*Cu', Q*Cz';
        Bw',                -gamma*eye(4),  Duw',                    Dzw';
        M*Bq'+X'*Bsigma'+Cu*Q', Duw,        -2*M,                    0;
        Cz*Q,               Dzw,              0,                     -gamma] <=0, Q>=0, M>=0]
```

Computational Example

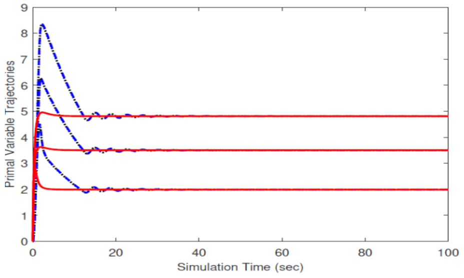
A quadratic programming problem:

minimize $\frac{1}{2}(4x_1^2 + 10x_2^2 + 3x_3^2 - 12x_1x_2 - 6x_2x_3 + 2x_1x_3)$
subject to $x_1 + 0.8x_2 + 1.2x_3 \geq 10$

Transforming this into a primal dual dynamic and simulating yields the following solution trajecories with and without antiwindup augmentation.



Without antiwindup



With antiwindup

Conclusion

An application that showcases the design is solving a quadratic program. These problems have extensive applications in control and the speed at which the quadratic program is solved is a vital component to performance. These involve the minimization of a multivariable expression as well as constraints. A systems ability to rapidly solve a quadratic program is indicative of its ability to perform control, so if the proposed anti-windup gain solves quadratic programs, it will also have control applications.

Future Work

Further work with this model would include implementing the model on a physical device. One possible candidate for the device could be the Quasar 2 DOF Helicopter. This system models air or underwater vehicles and would provide a reliable model to test all three controllers.

Contacts

Francis Moran
Email - moranf2@tcnj.edu
Cell – (732) 672 - 3049

Dr. Ambrose Adegbege
Email – adegbega@tcnj.edu